Data 8 Final Solutions

Summer 2017

First Name: ____

Last Name: ____

Email: ________________________________@berkeley.edu

Student ID: ____

GSI: ____

Name of person to your left: ________________________________

Name of person to your right: ________________________________

All the work on this exam is my own. **(please sign)**: ________________________

Instructions:

- This exam contains 17 pages (including this cover page) and 7 questions.
- This exam must be completed in the given **2 hour, 50 minute** time period.
- You may use two handwritten (two-sided) cheat sheets and the two official study guides provided with this exam.
- Work quickly through each question. There are a total of 107 points on this exam.

This page is intentionally left blank, but feel free to use it as scratch paper.

1. The happy table from Gallup World Poll data describes the happiness score ratings of 157 countries. It contains the number of people (in thousands) that gave each happiness rating (0 is worst, 10 is best) for each country.

```
Country | Rating | Count
Denmark | 0      | 3.4
Denmark | 1      | 4.5
...     | ...    | ...
Denmark | 10     | 16.9
Iceland | 0      | 2.1
Iceland | 1      | 4.1
... (1510 rows omitted)
```

Complete the Python expressions below to compute each result. The last line of each answer should evaluate to the result requested. **You may not use more lines than the ones provided.**

(a) (2 Pts) The total number of people surveyed in the data, in thousands.

np.sum(____)

Solution:

```
np.sum(happy.column('Count'))
```

(b) (3 Pts) A table containing the same columns as happy with the countries in descending order by the number of people in that country who gave a rating of 10. (All ratings in the table are integers.)

happy.____

Solution:

```
happy.where('Rating', 10).sort('Count', descending=True)
```

(c) (3 Pts) A table containing two columns: the countries and the total amount of people surveyed for each country in thousands.

happy.____

Solution:

```
happy.drop('Rating').group('Country', np.sum)
```

(d) (3 Pts) The average happiness rating of Denmark. (The answer is not simply 5. Remember to account for the rating counts.)

d = ____

sum(________________ * ________________) / sum(________________)

Solution:

```
d = happy.where('Country', 'Denmark')
sum(d.column(1) * d.column(2)) / sum(d.column(2))
```

(e) (6 Pts) A table containing the average happiness rating for each country. Assume you have a table called `totals` that contains the total people surveyed for each country:

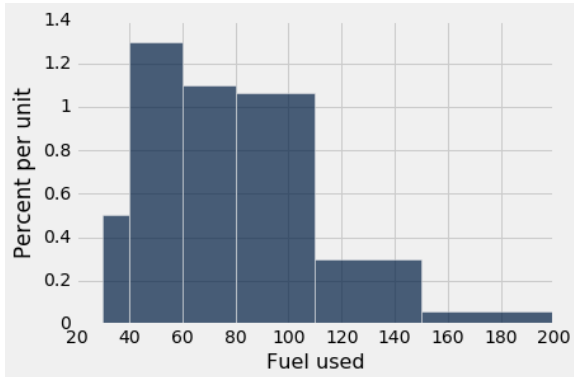
```
Country | Total
Denmark | 41.4
Iceland | 38.9
... (155 rows omitted)
```

```
def helper(row):
    total = totals.________________________________

    return ______________ * ______________ / __________

(happy.with_column('score', ________________________)
      .select('Country', 'score')
      .group('Country', np.sum))
```

Solution:

```
def helper(row):
    total = totals.where('Country', row.item(0)).column(1).item(0)
    return row.item(1) * row.item(2) / total

(happy.with_column('score', happy.apply(helper))
      .select('Country', 'score')
      .group('Country', np.sum))
```

2. This question uses the following histogram of the number of gallons of fuel consumed by different ships on the same trip.



The heights of the bars (in percent per gallon of fuel) are:

bin	[30, 40)	[40, 60)	[60, 80)	[80, 110)	[110, 150)	[150, 200)
height	0.5	1.3	1.1	1.07	?	0.06

(a) (3 Pts) Find the missing bin height in percent per gallon of fuel. You may leave your answer as an expression. If it is not possible to find the height, explain why.

Solution: The areas of the bars must sum to 100 percent, so:

$$\frac{100 - 10(0.5) - 20(1.3) - 20(1.1) - 30(1.07) - 50(0.06)}{40} \approx 0.296$$

(b) (3 Pts) Without any extra information, which bin has the most ships?

◯ [40, 60) ◯ [60, 80) ◯ [80, 110) ◯ Can't determine without additional information.

Justify your choice. If you picked the last choice, state the extra information you need.

Solution: The [40, 60) bar has an area of: $20 * 1.3 = 26\%$. The [60, 80) bar has an area of: $20 * 1.1 = 22\%$. The [80, 110) bar has an area of: $30 * 1.07 = 30.21\%$.

So, the [80, 110) bin has more ships regardless of how many ships there are total.

(c) (4 Pts) Suppose we merge the [30, 40) and [40, 60) bin together to create a single bin for [30, 60). What is the new height of [30, 60) bin (in percent per gallon)? You may leave your answer as an expression. If you need more information, state the information you need.

Solution: The [30, 40) bin has an area of: $10(0.5) = 5\%$. The [40, 60) bin has an area of $20(1.3) = 26\%$.

The total area is then 31%. Dividing by the total width of the new bin we have a height of: $31/(30) \approx 1.03$.

Note that you didn't need to know the number of ships in the [30, 40) bin to solve this problem!

3. A jar contains 100 jelly beans of four different colors - orange, green, blue, and purple - in equal amounts. **In this question, if you select the Other choice, you must state the correct expression in the space provided.**

   (a) (2 Pts) Suppose Sam only likes green jelly beans. He picks five beans at random without replacement from the jar. What is the chance he gets all green beans?

   √ $\frac{25}{100} \cdot \frac{24}{99} \cdot \frac{23}{98} \cdot \frac{22}{97} \cdot \frac{21}{96}$
   ◯ $\left(\frac{25}{100}\right)^5$
   ◯ $\frac{25}{100} \cdot \frac{24}{100} \cdot \frac{23}{100} \cdot \frac{22}{100} \cdot \frac{21}{100}$
   ◯ Other

   If Other selected, write correct expression:

   (b) (3 Pts) Suppose he replaces the five previous beans and picks another five, this time with replacement. What is the chance he gets at least one green bean?

   ◯ $1 - \left(\frac{25}{100}\right)^5$
   ◯ $1 - \frac{25}{100} \cdot \frac{24}{100} \cdot \frac{23}{100} \cdot \frac{22}{100} \cdot \frac{21}{100}$
   √ $1 - \left(\frac{75}{100}\right)^5$
   ◯ Other

   If Other selected, write correct expression:

   (c) (3 Pts) If Sam puts the beans back in the jar and then draws four beans without replacement, what is the chance that all four beans are different colors?

   ◯ $\frac{25}{100} \cdot \frac{24}{99} \cdot \frac{23}{98} \cdot \frac{22}{97}$
   ◯ $1 - \frac{25}{100} \cdot \frac{24}{99} \cdot \frac{23}{98} \cdot \frac{22}{97}$
   ◯ $\frac{25}{100} \cdot \frac{25}{99} \cdot \frac{25}{98} \cdot \frac{25}{97}$
   √ Other

   If Other selected, write correct expression:

   Solution:

   $$\frac{100}{100} \cdot \frac{75}{99} \cdot \frac{50}{98} \cdot \frac{25}{97}$$

   (d) (4 Pts) Sam receives a gift jar of 400 jellybeans that has 50 orange, 50 green, 150 blue, and 150 purple jellybeans. He's confused because he thinks the jar should have around the same proportion of each color, so he runs a hypothesis test.

   Null hypothesis: Each jellybean color has equal probability of appearing.

   Alternative: The jellybean colors appear with some other distribution.

   He selects the total variation distance as his test statistic.

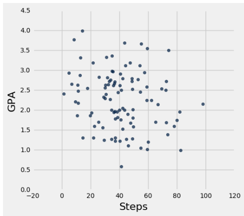
   Draw a rough sketch of the empirical distribution of the test statistic under the null hypothesis. You must:

   1. Include the origin (0, 0) in your plot.
   2. Mark the observed value of the test statistic as a vertical line.
   3. Shade in the area corresponding to the P-value.

   Don't worry about x/y labels or whether you shaded in exactly the right amount of area.

Solution: TODO(sam): Add plot.

4. Researchers collected data on the number of times students have stepped on a UC Berkeley seal in the past year and students' GPAs. A scatterplot of GPA on number of steps (left), as well as the table `seal` (middle) and its corresponding statistics (right), is displayed below. You may assume that statistics in that table are good approximations to their corresponding population parameters.



Student ID	Steps	GPA
25789012	65	2.86
38973744	39	3.76
31402938	96	3.12

`... (97 rows omitted)`

Statistic	Steps	GPA
Mean	50	2.5
Std Dev	20	1.5

(a) (4 Pts) Researchers want to create a confidence interval for the true slope of the regression line predicting GPA given the number of steps on the seal. Complete the code below so that the last line outputs a 90% confidence interval.

You may use the `slope(table, x_label, y_label)` function defined in class which takes in a table, the label/index for the x-values, and the label/index for the y-values, in that order. It returns the slope in original units of the regression line predicting y given x.

```
slopes = make_array()
for i in np.arange(5000):

    resample = ________________________________________

    bootstrapped_slope = ________________________________

    slopes = ______________ (slopes, __________________ )

[percentile(______, slopes), percentile(______, slopes)]
```

Solution:

```
slopes = make_array()
for i in np.arange(5000):
    resample = seal.sample()
    bootstrapped_slope = slope(resample, "Steps", "GPA")
    slopes = np.append(slopes, bootstrapped_slope)

print(percentile(5, slopes), percentile(95, slopes))
```

(b) (12 Pts) Assume the code above was implemented correctly and researchers generate an interval of $(-0.25, 0.12)$ for the population regression slope. Evaluate whether each statement is true or false and **briefly** justify your answer.

i. Because the interval contains 0, for a p-value cutoff of 5% we fail to reject the null hypothesis that the slope of the population regression line is 0.

√ True ◯ False

Solution: True. The confidence interval fails to reject this null hypothesis at a cutoff of 10%, which means we would also fail to reject the null at a cutoff of 5% (which gives a wider interval).

ii. If we rerun the code in part (a) 100 times, we expect that around 90 of the intervals will contain the true population slope.
◯ True √ False

Solution: False. We aren't getting a new sample from the population, so if we reran the code in part (a) we would get approximately the same interval every time.

iii. The population regression slope falls within this interval about 90% of the time.
◯ True √ False

Solution: False. The population regression slope either falls within this interval or it doesn't.

iv. If we instead try to predict number of steps using GPA, we will still get roughly the same confidence interval endpoints.
◯ True √ False

Solution: False. Although the interval will likely contain 0, the units of the interval will be different so the endpoints will also be different.

(c) (3 Pts) Suppose researchers are now interested in the average number of times students have stepped on the UC Berkeley seal in the past year. What sample size do they need to create a 95% confidence interval such that the interval width is 4 or less? If you need more information to solve this problem, state what information you need.

Solution: The interval width must be 4 or less and there are 4 standard deviations in a 95% confidence interval, so the standard deviation of the sample distribution must be 1 or less. Using the sample standard deviation as an approximation for the population standard deviation and applying the CLT, we get:

$$\frac{20}{\sqrt{n}} \leq 1$$
$$n \geq 400$$

(d) (5 Pts) Using the data from `seal`, researchers create a 95% bootstrap confidence interval for the average number of times students have stepped on a UC Berkeley seal in the past year. Which of the following statements is/are true about this interval? Select all that apply.

☐ If researchers draw another sample and computed a second confidence interval, it will always overlap with the first interval.

☐ If researchers increase their sample size from 100 to 1000, they will always get a smaller bootstrap confidence interval.

- ☐ If the researchers increase their bootstrap resample size from 100 to 1000, they will always get a confidence interval they can use to conduct a valid hypothesis test.
- ☐ If the researchers increase their bootstrap repetitions from 5000 to 10000, they can expect their confidence interval to be narrower.
- √ If the researchers resampled without replacement, the resulting interval will always have a width of 0.

Solution: Choice 1 is false. We are not guaranteed that the intervals will overlap since we could get a sample that looks completely different from our first one.

Choice 2 is false. We are not guaranteed that the new interval will be smaller, although it is highly likely.

Choice 3 is false. We must resample the same number of items as the original sample for our confidence interval to be valid.

Choice 4 is false. Increasing the bootstrap repetitions makes the empirical distribution of the test statistic smoother but does not significantly change the boundaries of the confidence interval.

Choice 5 is true. If we resample without replacement, we will get the same resampled slope every time.

(e) (4 Pts) Which of the following statements are true about the confidence interval created in part (a) and the confidence interval in part (d)? Select all that apply.

- √ Neither interval is guaranteed to contain the parameter it is trying to estimate.
- ☐ The interval in part (d) will be wider because it has a higher confidence level.
- ☐ The two intervals used different resample sizes.
- ☐ The interval estimating the population regression slope can be used to test a null hypothesis, but the one estimating average steps on the seal cannot.

Solution: Choice 1 is true. We are not guaranteed that either interval will contain the parameter.

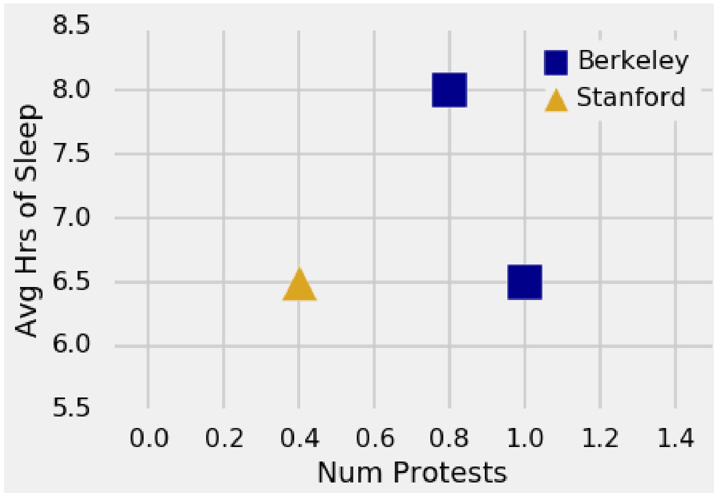
Choice 2 is false. The interval in (d) will be wider because it has different units, not because of its confidence level.

Choice 3 is false. We must resample the same number of items as the original sample for both confidence intervals.

Choice 4 is false. Both intervals can be used for hypothesis testing.

5. Emma thinks that students can be classified either as a Berkeley student or Stanford student by looking at each student's average number of protests attended per semester and their average hours of sleep per night.

   This scatter plot shows Emma's training set of three points for a nearest-neighbor classifier.



Note that the axes are not on the same units.

(a) (3 Pts) How would each unknown student be classified using a 1-nearest-neighbor classifier?

i. Student with 0.6 protests per semester and an average of 6.5 hours of sleep.
◯ Berkeley √ Stanford

ii. Student with 0.6 protests per semester and an average of 7.5 hours of sleep.
√ Berkeley ◯ Stanford

iii. Student with 0 protests per semester and an average of 7.5 hours of sleep.
√ Berkeley ◯ Stanford

(b) (3 Pts) Draw the decision boundary for a 1-nearest neighbor classifier on the scatterplot above.

(c) (3 Pts) Suppose Emma's test set contains 100 examples all labeled Berkeley that are distributed evenly throughout the area shown on the scatter plot above. Which classifier will have a higher **test set** accuracy when trained on her training set? Explain.

◯ A 1-nearest neighbor classifier √ A 3-nearest neighbor classifier

Solution: A 3-NN classifier will always predict Berkeley, which means it will have an prediction accuracy of 100%.

(d) (3 Pts) Suppose Emma trains a 3-nearest neighbor classifier on her training set. If we give the classifier a brand new point and it was classified correctly, what is the probability that the new point was originally labeled Stanford? Explain and show your work.

Solution: Since a 3-NN classifier always classifies points as Berkeley, if the classifier was correct the point was originally labeled Berkeley with probability 1. This means that the probability that the point was originally labeled Stanford is 0.

6. The `food` table contains the latitude and longitude of restaurants in SF and their food safety rating (0 to 100, higher is better).

```
lat       | long        | score
37.791116 | -122.403816 | 98
37.786848 | -122.421547 | 100
37.792888 | -122.403135 | 70
... (6723 rows omitted)
```

(a) Sam tried to do some data analysis but didn't get very far. Here are some variables he defined and how he defined them. The functions `slope(table, x_label, y_label)` returns the slope in original units of the regression line predicting y given x and `correlation(table, x_label, y_label)` returns the correlation coefficient r between the variables x and y.

Write one line of Python code to compute each of the following. **In your code, you may only use arithmetic operators, numbers, and the variables that Sam defined below.**

```
name | code
a1   | slope(food, 'lat', 'score')
corr | correlation(food, 'lat', 'score')
m1   | np.mean(food.column('lat'))
m2   | np.mean(food.column('score'))
```

i. (2 Pts) The correlation between score and latitude.

____

Solution:
`corr`

ii. (3 Pts) The prediction for the score given a latitude of 750.

____

Solution:
`a1 * 750 + m2 - a1 * m1`

iii. (3 Pts) The prediction for the latitude given a score of 0. (You don't need the next part of this question to solve this one.)

____

Solution:

```
m1 - corr * corr / a1 * m2
```

This question was taken out of the exam because you actually did, in fact, need part iv to solve this question.

iv. (3 Pts) The slope of the regression line predicting latitude given the score in original units.

____

Solution:

```
corr * corr / a1
```

(b) (5 Pts) The `food` table is repeated here for your convenience.

```
lat       | long        | score
37.791116 | -122.403816 | 98
37.786848 | -122.421547 | 100
37.792888 | -122.403135 | 70
... (6723 rows omitted)
```

Recall that the regression line is the line that minimizes the mean squared error between the predictions and the actual values. For example, to find the regression line that uses latitude to predict score we want to choose the slope and intercept for the line:

$$\text{predicted score} = slope \cdot lat + intercept$$

So that $mean((\text{actual score} - \text{predicted score})^2)$ is as small as possible.

We can use the `minimize` function to find this slope and intercept for us. In particular, in lecture we defined a function `mse(slope, intercept)` that returns the mean squared error given a choice of slope and intercept. Then, `minimize(mse)` returned the slope and intercept of the regression line.

What if we wanted to use both latitude and longitude to make a prediction? We can do this by minimizing the mean squared error between the prediction line

$$\text{predicted score} = slope_1 \cdot lat + slope_2 \cdot long + intercept$$

And the actual score. This is also known by the term *multiple linear regression*.

Fill in the the blanks for `multiple_mse` so that `minimize(multiple_mse)` will return the three parameters for linear regression using both latitude and longitude to predict score.

```
def multiple_mse(________________________________________):
    lat = ____________________________________________
    long = ___________________________________________
    score = __________________________________________
    return np.mean(
    __________________________________________________
    )
```

Solution:

```
def multiple_mse(slope1, slope2, intercept):
    lat = food.column('lat')
    long = food.column('long')
    score = food.column('score')
```

```
    return np.mean(
      (score - slope1 * lat - slope2 * long - intercept) ** 2
    )
```

7. Arvind is testing the effectiveness of a drug, Oskirol, on patients who suffer from the worldwide Tree Syndrome epidemic. He randomly selects 50 patients from the Junior University Hospital, then randomly assigns half of the patients to a treatment group and half to control. He gives the treatment group the drug and doesn't do anything to the control group.

   Afterward, Arvind calculates an overall wellness score from 0 to 100 for all patients. He creates a 95% bootstrap confidence interval for the difference in the mean wellness scores between treatment and control groups: [11.1, 18.4].

   Sam has some complaints about Arvind's methods. Evaluate whether Sam is right in each scenario and justify your answer.

   (a) (3 Pts) Sam states that since Arvind only selected patients from the Junior University Hospital, Arvind cannot infer any effect his treatment has on **all** patients with Tree Syndrome.

   √ Sam is correct ◯ Sam is incorrect

   Solution: Sam is correct. If Arvind only drew patients from the Junior University Hospital, with no other information he can only infer the effect his treatment has on patients with Tree Syndrome at that hospital.

   (b) (3 Pts) Sam finds out that the true probability distribution of wellness scores is highly skewed left. Because of this, he states that the bootstrap is an **inappropriate method** to use.

   ◯ Sam is correct √ Sam is incorrect

   Solution: Sam is incorrect. According to the Central Limit Theorem, the distribution of the mean wellness scores is normally distributed even though the actual distribution of scores is not, so the conditions for bootstrapping are still met.

   (c) (3 Pts) Arvind notices that 0 is not in the confidence interval, so he concludes that is it likely his drug **caused** his patients' wellness scores to increase. Sam points out that Arvind's conclusion is incorrect: this hypothesis test can only be used to establish association, not causation.

   ◯ Sam is correct √ Sam is incorrect

   Solution: Sam is incorrect. Because this experiment was a randomized control experiment, if we reject the null we can establish causality.

   Note that Arvind can conclude that his drug likely caused wellness scores to increase compared to patients who didn't take anything at all. He can't say anything about the effectiveness of his drug compared to a placebo which means this experiment needs adjusting for actual medical purposes.

   (d) (3 Pts) Sam points out that Arvind didn't give a placebo to the control group, so the control group's wellness scores were not as high as they would be with a placebo. Sam argues that Arvind's confidence interval endpoints are likely **lower** than they would be with the placebo.

   ◯ Sam is correct √ Sam is incorrect

   Solution: Sam is incorrect. Since we expect a placebo to increase the wellness scores of the control group, Arvind's confidence interval endpoints are likely higher that the one he would have created if he gave a placebo to the control group.

This page is intentionally left blank, but feel free to use it as scratch paper.